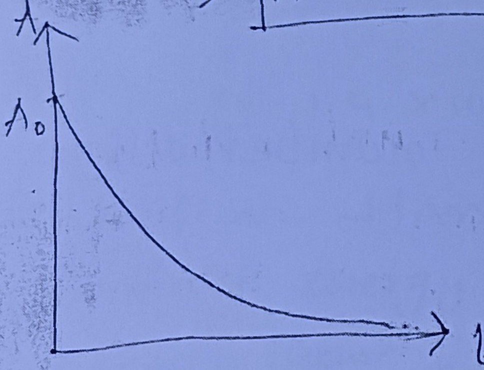
... formu

$\Rightarrow \lambda N = \lambda N_0 e^{-\lambda t}$

$\Rightarrow \boxed{A = A_0 e^{-\lambda t}}$

A = activity at time 't'.

A_0 = activity at time 't = 0'



unit → Bq (Bequerel)

1 Bq = 1 DPS [Disintegration per second]

(*) Half life of C^{14} is 12 years. find number of C^{14} atom present, after 10 year, if initially 10 gm of C^{14} is present.

→ $N_0 = 10$ gm

$t_{1/2} = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{12} = 0.05775$

$\therefore N = 10 e^{-0.05775}$

$\Rightarrow N = 10 \times 0.5612$

$\Rightarrow N = 5.612$ gm

14 gm ⟶ 6.022×10^{23}

$\therefore$ 1 " ⟶ $\frac{6.022 \times 10^{23}}{14}$

$\therefore$ 5.612 gm ⟶ $\frac{6.022 \times 10^{23} \times 5.612}{14}$

$= 2.413 \times 10^{23}$ atom's

(*) i) Discuss about different types of nucleus.

ii) Find the relation between radius of atom and mass number. Hence find the density of nucleus.

→ (i) isotope :- Same no. of proton / atomic no. and different mass no. $_6C^{12}$, $_6C^{14}$

isobar:- Same mass number and differed atomic number / Proton number.

$_7N^{14}$, $_6C^{14}$.

isotone:- Same nutron number diffrent Proton no.

$_7N^{14}$, $_8O^{15}$.

Mirror nucleus:- The nutron no. of 1st nucleus, is equal to Proton number of 2nd nucleus and Proton no of 1st nucleus equal to nutron no. of 2nd nucleus.

$_9F^{17}$, $_8O^{17}$

$P = 9$ ⟶ $P = 8$

$N = 8$ ⟵ $N = 9$

(ii) The volume of nucleus ∝ mass no.

$$\frac{4}{3}\pi R^3 \propto A$$

$$\Rightarrow R^3 \propto A$$

$$\Rightarrow \boxed{R = R_0 A^{1/3}}, \quad R_0 = 1.4\,fm \;\; A<30$$

$$= 1.2\,fm \;\; A>30$$

This is the relation between radius of the nucleus and the mass number of the nucleus.

$$\text{density } (\rho) = \frac{\text{Mass}}{\text{volume}}$$

$$= \frac{A \times m_p}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \frac{A\, m_p}{\frac{4}{3}\pi R_0^3 A}$$

$$= \frac{3 m_p}{4\pi R_0^3} \qquad m_p = 1.67 \times 10^{-27}\, kg$$

$$R_0 = 1.4 \times 10^{-15}\, m$$

$$\simeq 1.4 \times 10^{17}\, kg/m^3$$

(*) Define Mass defect of a nucleus and binding energy of nucleus. Find the relation between them?

→ The difference in mass of a nucleus and it's constituent nucleon's is called the mass defect of that nucleus.

$$\Delta M = Z m_p + (A - Z) m_n - M \text{ (mass of nucleus)}$$

The Energy equivalent of the mass defect of a nucleus is called its binding energy.

$$BE = \Delta M c^2$$

(*) Draw the graph of binding Energy per nucleon Vs mass number graph. Hence Discuss Different information obtain from this graph?

→

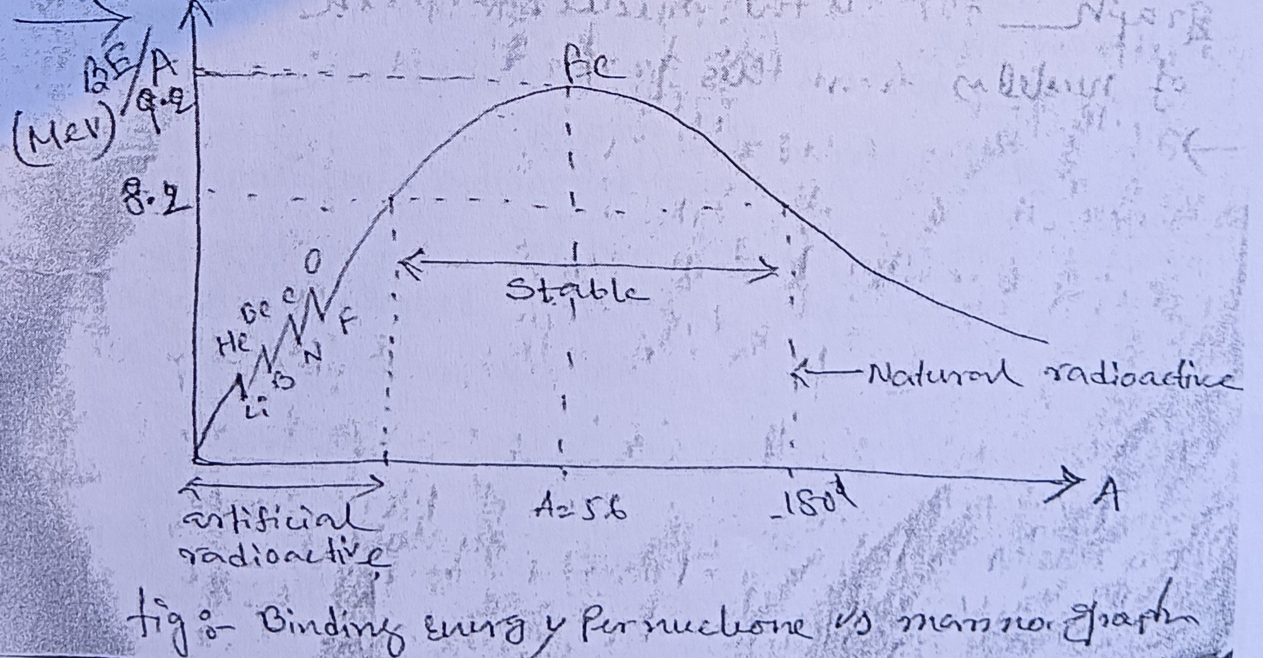


fig:- Binding energy Per nucleone vs mass no. graph

2/3

* Using semi-empirical mass formula find the atomic number of most stable isobar.

→ Isobar → same mass no. different atomic no.

from, semi-emperical mass formula,

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} \pm \delta(A)$$

$$\frac{\partial BE}{\partial Z} = -\frac{a_c}{A^{1/3}}(2Z-1) + 4a_{as}\frac{(A-2Z)}{A}$$

$$\Rightarrow 0 = -\frac{a_c(2Z-1)}{A^{1/3}} + 4a_{as}\frac{(A-2Z)}{A}$$

$$\Rightarrow 0 = -\frac{2a_c Z}{A^{1/3}} + \frac{a_c}{A^{1/3}} + 4a_{as} - \frac{4a_{as} 2Z}{A}$$

$$\Rightarrow Z\left(\frac{2a_c}{A^{1/3}} + \frac{8a_{as}}{A}\right) = \frac{a_c}{A^{1/3}} + 4a_{as}$$

$$\Rightarrow Z = \frac{a_c A^{-1/3} + 4a_{as}}{2a_c A^{-1/3} + 8a_{as} A^{-1}}$$

$$\Rightarrow \boxed{Z = \frac{4a_{as} A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_{as}}}$$

this is the expression for most stable isobar

(*) Using semi-empirical mass formula, find the binding energy of (i) $_{20}Ca^{40}$, (ii) $_{19}K^{39}$

→ given $a_v = 15.5$ Mev, $a_s = 16.8$ Mev, $a_c = 0.7$ Mev

$a_{as} = 23$ Mev, $a_p = 34$ Mev.

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} + (0, \pm a_p) A^{-3/4} - a_{as} \frac{(A-2Z)^2}{A}$$

(i) $Z = 20$, $N = 20$,

Pairing energy, $+a_p$.

$$\therefore BE = 15.5 \times 40 - 16.8 \times (40)^{2/3} - 0.7 \times \frac{20 \times 19}{(40)^{1/3}} + 34(40)^{-3/4} - 23 \times 0.$$

$$= 620 - 16.8 \times 11.84 - 6.65 + \frac{34}{15.905}$$

$$= 620 + 2.137 - 205.562$$

$$= 416.575 \text{ Mev.}$$

(ii) $Z = 19$, $N = 20$

Pairing energy $= 0$

$$\therefore BE = 15.5 \times 39 - 16.8 \times (39)^{2/3} - 0.7 \times \frac{19 \times 18}{(39)^{1/3}} + 0 - \frac{23}{39}$$

$$= 604.5 - 195.589 - 6.138 - 0.589$$

$$= 402.184 \text{ Mev}$$

(*) Find the most stable isobar for mass number A = 56.

$a_v = 15.5$ Mev, $a_s = 16.8$ Mev, $a_c = 0.7$ Mev, $a_{as} = 23$ Mev, $a_p = 34$ Mev.

Atomic number for most stable isobar

$$Z = \frac{a_c A^{2/3} + 4a_{as}A}{2a_c A^{2/3} + 8a_{as}}$$

$$= \frac{0.7\times(56)^{2/3} + 4\times 23\times 56}{2\times 0.7\times(56)^{2/3} + 8\times 23}$$

$$=$$

$$= 25.24$$

$\simeq 26$ $\,_{26}Fe^{56}$ is most stable isobar corresponding to mass no. 56.

write down the evidance of nuclear shell model.

(*) In nuclear shell Model, we consider that Proton's, nutron's form their individual shell's or orbit like structure similar to electron configuration.

For the closed shell of Proton and neutron, the nucleus is more stable compare to nucleus with half filled or less than half filled shell.

There are many physical proves of nuclear shell model —

(i) Some nucleuses with particular

no. of Proton and nutron is more stable, this number's are known as magic no.

Magic no.'s are – 2, 8, 20, 50, 86, 126. This magic no.'s cannot be explain using liquid drop model.

(ii) nucleus with magic no. proton's has high separation energy for proton as compared to other nucleuses in the naighbourhood.

(iii) nucleus with magic no. nutron's has high separation energy for nutrons as compared to other nucleuses in the naighbourhood.

(iv) Proton capture cross section or proton capture probability of a nucleus with magic no. proton's is very small as compared to naighbourhood nuclei. because the proton shell is closed for this nucleus.

(v) natron capture cross section or nutron capture probability of a nucleus with magic no. ~~proton~~ nutron's is very small as compared to neighbourhood nuclei. because the nutron shell is closed for this nucleus.

(vi) in liquid drop model it was consider that, the shape of the nucleus is sperical but in practical only the nucleuses with

magic no. Proton and nutron has spherical shape., other nucleus es has shape Prolet and oblet.

(*)(i) Write down the advantage's of nuclear Shell model over liquid drop model?

(*)(ii) Write down the limitation's of nuclear Shell model.

(*)(6) Explain the origin of magic no's with mayer correction.

(OR)

Explain how magic no's represent closed Proton nutron shell.

(OR)

With spin orbit interaction of Proton nutron explain the magic no's,

(i) → Advantage's of nuclear shell model :-

(1) It explain's very well the existence of magic number's and the stability and high binding energy on the basis of closed shells.

(2) The shell model provides explanation for the ground state spins and magnetic moments of the nuclei.

(3) Nuclear isomerism, i.e existance of isobaric, isotopic nuclei in different energy state's of odd-A nuclei between 69-81, 111-125 has been

explained with shell model.

i) → Limitation's of nuclear shell model are:-

(1) The model does not predict the correct value of spin quantum number in certain nuclei. Ex- $^{23}_{11}Na$ where the predicted value is $5/2$, the corrected value is $1/2$

(2) The following four stable nuclei $^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}B$, $^{14}_{7}N$ do not fit into this model.

(3) The model cannot explain the observed 1st excited state's in even-even nuclei at energies much lower than those expected from single particle excitation.

And also failed to explained the observed quadrupole moment of odd-A nuclei, in particular of those having A-values far away from the magic number.

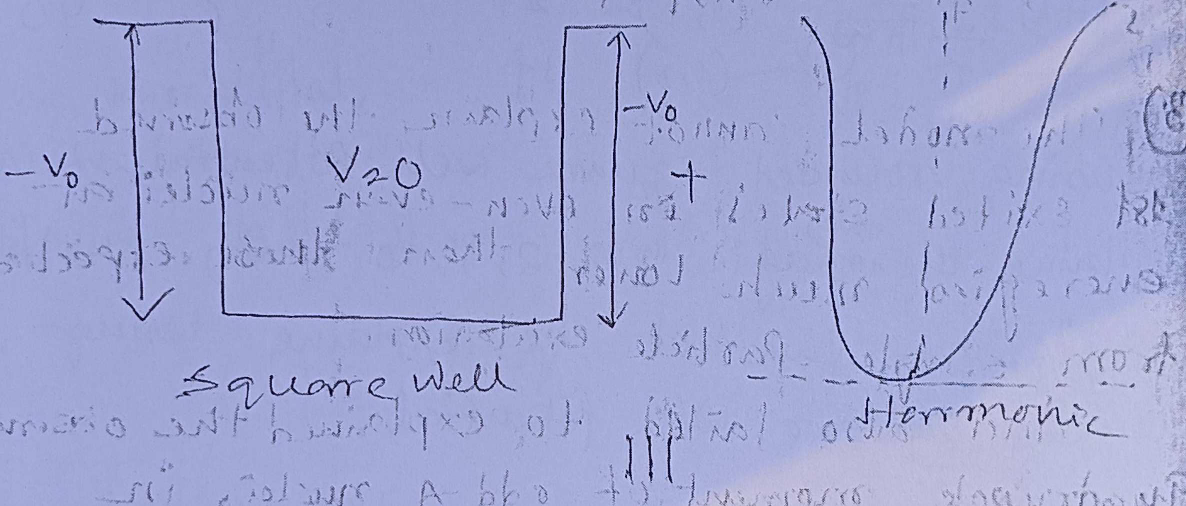
(5) ⊛ → By using schrodinger equ^n we can describe the motion of nucleon and different energy level of nucleon in a nucleous. But we donot know the exact profile of nuclear potential energy. (corrosponding to nuclear force)

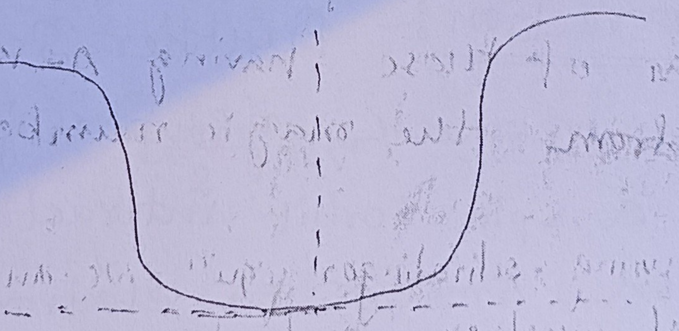
So, we have to use Heat and trial method for nuclear potential energy. Scientist try with different potential energy like square well potential, otropic Harmonic potential, wood-saxon potential

$$\left[V(r) = \frac{V_0}{1 + e^{(r-R)/\delta}} \right]$$

Yukava Potential $\left[V(r) = \frac{V_0 e^{-\mu r}}{r} \right]$. rounded square well Potential etc.

The best result was obtained by using rounded square well potential. rounded square well potential is a hybridised potential of square well potential and harmonic Potential.





rounded square well

Using rounded square well Potential in Schrodinger's eqn we get the energy level of nucleon's, $E_\lambda = (\lambda + \frac{3}{2})\hbar\omega$, where, $\lambda = 0, 1, 2, 3, \ldots$

where, n = quantum no. for square well potential, $\lambda = 2n + l - 2$,

l = quantum no. for isotropic harmonic Potential. $n = 1, 2, 3, \ldots$, $l = 0, 1, 2, 3 \ldots$

λ	Energy level $E_\lambda = (\lambda + \frac{3}{2})\hbar\omega$	(n, l)	nl (state)	2(2l+1) no. of nucleon	Magic no.
0	$\frac{3}{2}\hbar\omega$	(1, 0)	1s	2	2 ✓
1	$\frac{5}{2}\hbar\omega$	(1, 1)	1p	6	8 ✓
2	$\frac{7}{2}\hbar\omega$	(2, 0) (1, 2)	2s 1d	2 10 }	20 ✓
3	$\frac{9}{2}\hbar\omega$	(2, 1) (1, 3)	2p 1f	6 14 }	40

by using rounded square well potential we can Explaine the only magic no 2, 8, 20 only. further magic no. can not be Explaine using rounded square well potential.

Mayer Correction:- Further Scientist mayer and his research group sugested that their is a spin-orbit interaction of Proton and nutron similler to electron.

the spin-orbit interaction energy.

$$V(r) = -\phi(r)\,[\vec{L}\cdot\vec{S}] \quad \text{--- (1)}$$

$\phi(r)$ = co-efficient depends on r

total angular momentum

$$\vec{J} = \vec{L} + \vec{S}$$

$\vec{L}$ = orbital angular momentum.

$\vec{S}$ = spin angular momentum.

$$\Rightarrow J^2 = (\vec{L} + \vec{S})\cdot(\vec{L} + \vec{S})$$

$$\Rightarrow J^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$$

$\Rightarrow J^2 - L^2 - S^2 = 2\vec{L}\cdot\vec{S}$

$\Rightarrow \vec{L}\cdot\vec{S} = \dfrac{J^2 - L^2 - S^2}{2} \longrightarrow (ii)$

- Using equ^n (ii) in equ^n (i).

$V(r) = -\phi(r)\left[\dfrac{J^2 - L^2 - S^2}{2}\right]$,

$= -\phi(r)\dfrac{[j(j+1) - l(l+1) - s(s+1)]\hbar^2}{2}$

$J = \sqrt{j(j+1)}\,\hbar$

$L = \sqrt{l(l+1)}\,\hbar$ $\longrightarrow (iii)$

$S = \sqrt{s(s+1)}\,\hbar$

We know that,

$j = |l+s|, \ldots\ldots, |l-s|$

$\therefore j = l + \frac{1}{2},\ l - \frac{1}{2}$ for Proton, nutron $s = \frac{1}{2}$

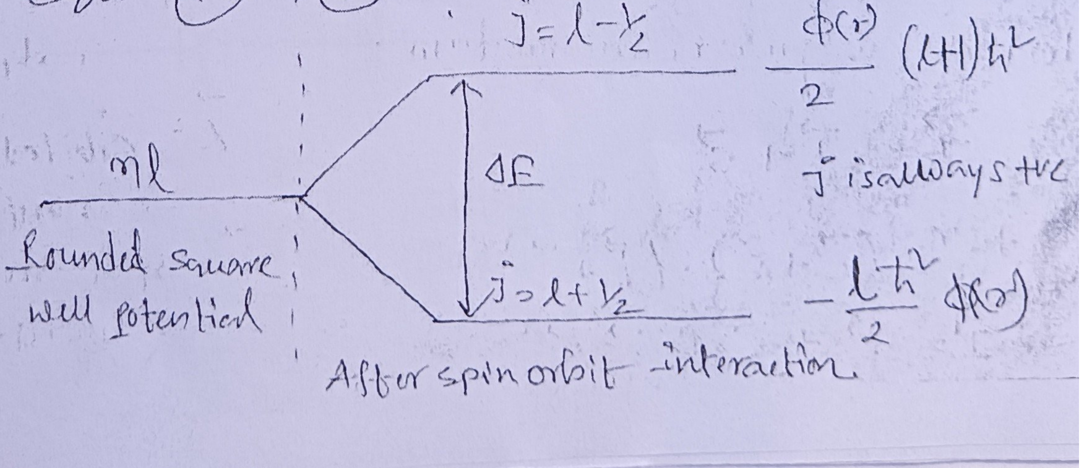
for, $j = l + \frac{1}{2}$, $s = \frac{1}{2}$

$V(r) = -\dfrac{l\hbar^2}{2}\phi(r) \longrightarrow (iv)$

for, $j = l - \frac{1}{2}$, $s = \frac{1}{2}$;

$V(r) = \dfrac{\phi(r)}{2}(l+1)\hbar^2 \longrightarrow (v)$

from equ^n (iv), (v)



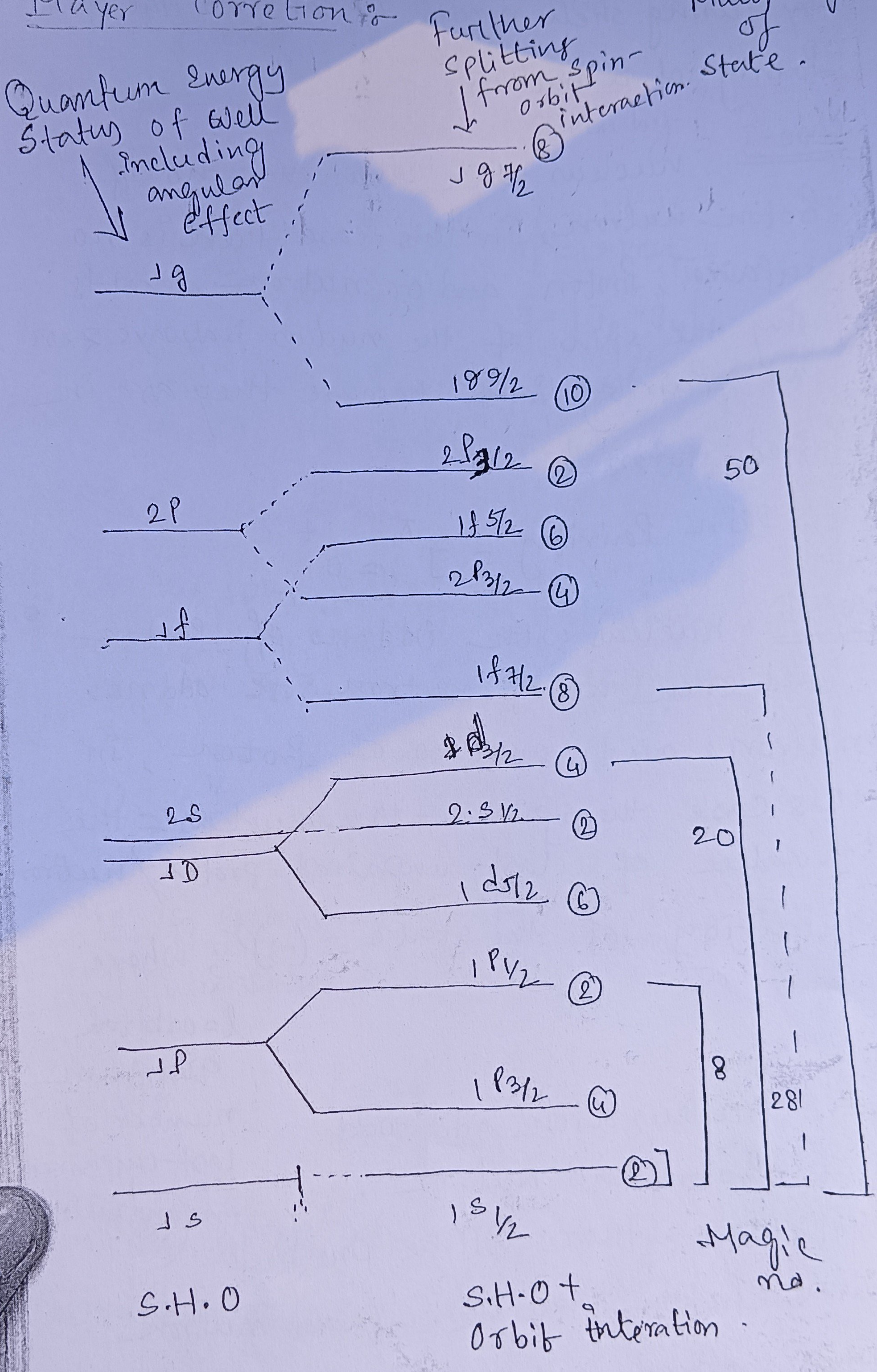
Application of sh...

the energy gap $\Delta E = \frac{\phi(r)\hbar^2}{2}[2l+1]$

i.e. $\Delta E \propto (2l+1)$

Note:- for every j value, no. of proton or neutron in each state $= (2j+1)$.

Energy level digram of shell model after Mayer corretion :-



Application of shell model :-

By using shell model we can find the spin pairity of a nucleus.

Case-I. nucleus with even even no. of proton-nutron, In this case there is no unpaired, proton ~~and~~ or nutron, that is why the spin of the nucleus is always zero and pairity is '+' because they are in paired form.

$$\text{Spin Pairity} = j^{\pi} = 0^{+}$$

Case-II. nucleus with odd-no. of proton and even no. of nutron ~~or~~ odd no. nutron and even no. of proton, in this case the spin of the nucleus = the 'j'-value of last unpaired proton/nutron the pairity of the state $= (-1)^l$; where l = orbital quantum number of last unpaired proton/nutron

Case-III. nucleus with odd-odd no. of proton and nutron, in this case, there will be one unpaired proton and one unpaired nutron.

for the unpaired proton the last energy level $= l_p, j_p$

for the unpaired nutron the Last Energy level = l_n, j_n

the spin of the nucleus is given by Nordhern rule,

if $|l_p + j_p + l_n + j_n| =$ even

the spin of the nucleus $j = |j_p - j_n|$

if $|l_p + j_p + l_n + j_n| =$ odd.

the spin of the nucleus $j = |j_p + j_n|$

Pairity $= (-1)^{l_p + l_n}$

⊛ Find the spin Pairity of following nucleuses:- (i) ${}_6C^{12}$, (ii) ${}_{19}K^{39}$, (iii) ${}_9F^{18}$ (iv) ${}_{19}K^{40}$

⟶ (i) ${}_6C^{12}$, $P = 6$

$n = 6$.

as there is even-even no. of Proton and nutron, spin-Parity $= 0^+$

(ii) ${}_{19}K^{39}$, $P = 19$

$n = 20$

$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2})^3$

for, unpaired proton, $j = 3/2$, $l = 2$

$\therefore$ Parity $= (-1)^{l} = (-1)^2 = +1$

N = No

Spin-Parity of this nucleus $= 3/2^{+}$

(iii) $_{9}F^{18}$, $p = 9$
$n = 9$.

for neutron, $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

$j_n = 5/2$, $l_n = 2$.

for Proton,
$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

$j_p = 5/2$, $l_p = 2$

using Nordheim rule,
$|l_p + j_p + l_n + j_n| = |5/2 + 2 + 5/2 + 2|$
$= 9$ odd.

∴ the spin of nucleus $= j = 5$

Parity $= (-1)^{4} = +1$

∴ Spin-Parity of nucleus $= 5^{+}$

(iv) $_{19}K^{40}$, $p = 19$
$n = 21$

For Proton, $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2})^3$

$j_p = 3/2$, $l_p = 2$

for neutron, $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (2d_{3/2})^4 (1f_{7/2})^1$

$j_n = 7/2$, $l_n = 3$.

Using Nordheim rule,

$|l_p + j_p + l_n + j_n| = 10$ even

$\therefore j = |-2| = 2$

$\therefore$ Parity $= (-1)^5 = -1$

$\therefore$ Spin-parity of nucleus $= 2^-$ Ans

(*) What do you mean by "Mean field theory" nucleus?

(OR)

Explain "Mean Field theory"

→ In atom the electron's are revolving arround the nucleus due to electrostatic interaction between the nucleus and electrons. But in case of proton and nutron's in nucleus, there is no such centre about which the proton and nutron can revolve. Scientist's assumed that the proton and neutron's experience's say nuclear force, which is the average of nuclear force of the whole nucleus (for 1 nucleon) and they revolve arround this mean force field of nuclear force. In this condition, we can describe the nature of whole nucleus by one nucleon only as each of the nucleon experiences

same force. This is known as "mean Field theory" of nucleus.

Mean field theory is used in single particle shell model.

⊛ Write down the assumption of fermi gas model of nucleus, write down it's merits and limitations

→ fermi gas Model

assumptions:-

(1) All Fermion's Occupy the lowest Energy states.

(2) Protons and nutron's are independent fermion filling two seperate potential well's.

(3) Common fermi Energy for Proton's and nutron's is stable nuclei.

2019

(1) → Packing fraction:- The ratio of the mass defect (mass difference between individual mass and ~~total~~ mass of nucleus) and the mass number of a nuclei is known as Packing fraction.

(2) → mass defect:- The difference in mass ~~and~~ of a nucleus and it's constituent nucleons is called the mass defect of that nucleus.

(5) → The names of various models of the nucleus is given by —

(i) Liquid drop model.
(ii) shell model.
(iii) compound nucleus model. (nuclear)
(iv) Fermi gas model of nucleus.

(6) → Because, As the no. of Proton increases in a nucleus, the coulomb's repulsive force increasing, which tends to break the nucleus apart. So, to keep the nucleus stable, more no. of neutrons are needed, which are neutral in nature.

(7) → The liquid drop model admits only a nuclear surface and interior. In contrast, shell models predicts energy shell and subshells and therefore admit rich internal ~~substru~~ substructure.

(8) → For the stability of nucleus some times neutron's are converted to proton and some times proton converted to neutron, when β-Particle emitt's from the nucleus.

... of synchrotron over cyclotron is

(9) → As the thickness of the material is increased the fraction of the radiation passing through the material will decrease.

(10) → Particle accelerator is a divise used to accelerate different charge particle's to high velocity or high KE, which is necessery for many nuclear reaction's or experiment's.

No, we can't accelerate neutrons by cyclotron.

(11) → (i) Initial energy of α particle.

(ii) The range of α particle inversely proportional to the inonization energy of the gas.

(iii) The range of α particle is inversely proportional to density of the medium.

(iv) The range of α particle proportional to temperature and inversely proportional to pressure.

(13) → Isospin as an abstract quantity, it is not physical quantity. In strong interaction between two particle's is independent of charge. To distinguigh between nutron - proton we consider a quantum no. which is known as isospin.

(12) → Cherenkov radiation is a form of energy that we can perceive as a blue glow emitted when the electrically charged particles that composes atom's are moving at speeds faster than that of light in a specific medium.

(4) → The angular momentum of the nucleus is the combined contribution of the spin-orbit angular momenta of the constituent particles.

nula,

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